covered with a semi-infinite layer of gyrotropic plasma.

In conclusion, it is pointed out, that a comprehensive treatment of the surface waves on a perfectly conducting screen covered with an anisotropic plasma sheath, is given for one simple orientation of the external magnetic field. The results of this paper are believed to provide an interesting extension to the results obtained by Tamir and Oliner [3] for the isotropic case.

Acknowledgment

The authors are grateful to Prof. R. W. P. King for help and encouragement with this research.

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[10] S. R. Seshadri and A. Hessel, "Radiation from a source near a plane interface between an isotropic and a gyrotropic dielectric," Applied Research Laboratory, Sylvania Electronic Systems, Waltham, Mass., Research Rept. No. 339; May, 1963.
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Back-Scattering Measurements of a Slowly Moving Target

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Summary—A basic problem in the measurement of back-scattering cross sections is the separation of the desired target-scattered signal from the undesired background reflections. An additional problem may be the separation from the target-scattered signal of signals directly coupled from the transmitter to the receiver. Historically, these have been overcome in several ways: 1) a reference signal has been used to cancel the undesired signals when measuring a fixed target, 2) a reference signal has been used to override the undesired signals when measuring a rapidly moving target, and 3) an average curve has been fitted to data taken with a target at several positions.

Two useful alternative techniques are described herein. A cancellation procedure performed while the target is slowly moving is shown to be effective in a much poorer environment than the static nulling procedure. The use of a reference signal to override the undesired signals is shown to be directly applicable to a slowly moving target procedure, thus simplifying the mechanical problems in measuring bulky targets. With a simple experimental setup, back-scattering cross sections 33 db below a square wavelength at 11 Gc can be measured at a range of 150 cm when transmitting 400 mw. These readings can be taken in an environment 20 to 30 db worse than that usually considered necessary for scattering measurements by the static null procedure.

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Introduction

The theoretical determination of the back-scattering cross section of any except simple symmetrically shaped objects is exceedingly difficult because of mathematical complexities. Even with simple shapes, it is satisfying to check theory with experimental results. Thus a number of experimental procedures have been developed for measuring the back-scatter cross section of objects with complex shapes.[1–7]

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[3] H. Scharfman and D. D. King, "Antenna-scattering measurements by modulation of the scatterer," Proc. IRE, vol. 42, pp. 854–858; May, 1954.

[4] R. W. P. King and T. T. Wu, "The Reflection of Electromagnetic Waves from Surfaces of Complex Shapes," Cruft Lab., Harvard Univ., Cambridge, Mass., Sci. Rept. No. 12; November, 1957.

[5] H. J. Schmitt, "Back-scattering measurements with a space-separation method," IRE Trans. on Antennas and Propagation, vol. AP-7, pp. 15–32; January, 1959.

[6] C. C. H. Tang, "Electromagnetic backscattering measurements by a time-separation method," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 209–213; April, 1959.

[7] E. B. McMillan and H. J. Schmitt, "Doppler method for absorber testing," *Microwave J.*, vol. 3, pp. 64–68; November, 1960.

The approach usually taken is to transmit a wave toward the scatterer and measure the scattered wave returning. A basic problem is the separation of the wave scattered by the object being studied from that scattered by its surroundings. Depending on the experimental approach, there may be an additional problem of separating the scattered waves from the wave coupled directly to the receiving antenna. Ideally, the first problem would be eliminated by having a perfect environment; *i.e.*, one with no scattering obstacles except the one being measured. Since it is impossible to do this in the practical situation, background scattering becomes one of the most important problems. The nulling procedure[2,5] and the Doppler procedure[3,7] attempt to overcome the effects of both types of unwanted received signal by the use of a controlled reference signal. In the nulling procedure the reference signal is adjusted to cancel the undesired signals while in the Doppler procedure it is used to override them. The Doppler frequency shift of the target-scattered signal allows it to be separated, while in the nulling procedure the target signal is the only one left. The nulling technique is seriously affected if the target forward scatter or "shadow effect" causes the background scattered wave to change when the target is inserted.

Although the Doppler effect is normally thought of in terms of frequency shift, the phenemenon continues as the relative velocity of source and observer becomes less and less. It eventually becomes more lucid to think of the effect as a slow phase shift of a CW signal. This varying phase allows the returning wave to be separated, in principle, just as easily as if it were a frequency shift.

The two simple procedures described in this report make use of the phase shift in the signal received as the target is moved very slowly. In fact, constant velocity motion is not necessary. A dynamic nulling technique is described that has an advantage over the conventional nulling technique in that the "shadow effects" of the target on the background may be ignored since nulling is accomplished while the target is in place. This also means that frequency drift of the source is not a serious problem. Another technique is described wherein a high-level reference signal is used to override the background effects as per conventional Doppler setup. The slowly moving target has the advantage of mechanical simplicity when compared to the conventional Doppler setup. In both modes of operation, there are no target symmetry requirements, and a ground plane is not necessary.

Measurement Apparatus

A block diagram of the measuring setup is shown in Fig. 1. The directional coupler provides a sample of the transmitted signal which is controlled in magnitude and phase. This reference signal and the received signals are combined in the matched hybrid-T and applied to the untuned crystal detector. This is recognized as basically a two-antenna nulling circuit. In the second mode of

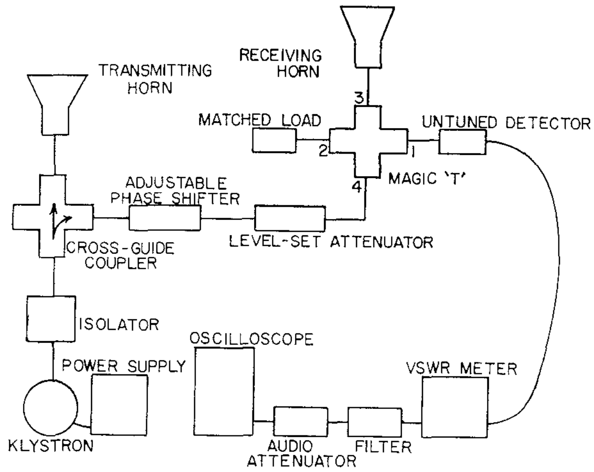

Fig. 1—Block diagram of measuring apparatus.

operation, the phase shifter is not necessary and could be removed to give what is basically a two-antenna Doppler circuit.

The electric field applied to the crystal detector input waveguide is the sum of the reference signal and the components from the target and background.[8] Assuming that the background signal is not dependent on the target and that the target signal amplitude is not dependent on the range R, the electric field at the input to the crystal detector can be written, in phasor form,[9]

$$E_D\epsilon^{j\theta_D} = E_{rb}\epsilon^{j\theta_{rb}} + E_t\epsilon^{j(\theta_t+2kR)} \tag{1}$$

where

$$E_{rb}\epsilon^{j\theta_{rb}} = E_r\epsilon^{j\theta_r} + E_b\epsilon^{j\theta_b}. \tag{2}$$

Assuming the crystal detector has a square-law characteristic

$$i_D \,\alpha\, e_D^2 \tag{3}$$

the output becomes

$$V_{out} = K[E_{rb}^2 + E_t^2 + 2E_tE_{rb}\cos(2kR + \theta_t - \theta_{rb})]. \tag{4}$$

Eq. (4) consists of two dc terms plus a cosine term which goes through a complete cycle as R varies $\lambda/2$. The variation with R is independent of the velocity of the target.[10]

In the experimental setup, the klystron is modulated with a 1000 cps square wave. The audio signal is amplified in the tuned amplifier which has a rectifier in its output circuitry. Thus, after amplification, the

[8] The term E_b, "background signal," is used somewhat loosely here and in the following material to indicate the sum of the undesired signals not considered explicitly.

[9] The symbols E_D, E_r, E_t E_b refer respectively to the electric fields of the detector, reference, target, and background, all at the detector input guide.

[10] If the target is allowed to move with a constant velocity v, the variational component of the detector output voltage becomes the expected Doppler term

$$KE_tE_{rb}\cos\left[\omega\left(\frac{2v}{c}\right)t + \theta_t - \theta_{rb}\right]$$

where ω is the frequency of the transmitted signal and c is the velocity of light.

rectified 1000 cps output is given by an equation which is identical to (4) but where "K" now includes amplifier gain and the rectifier characteristics. An oscilloscope can be used to observe the rectified and filtered output. The horizontal deflection of the oscilloscope is made proportional to R with a potentiometer connected to the drive shaft of the moving target setup. The audio attenuator can be used to read relative levels, keeping the oscilloscope deflection the same.

The targets are placed on a polyfoam column attached to a carriage which rolls on metal tracks. The carriage is driven at a velocity of approximately $\frac{1}{4}$ cm per second by an induction motor.

Modes of Operation

In (4) it was assumed that the target signal was independent of the range R. Actually the power received from the target is shown by the radar equation to vary as $1/R^4$ so that in a given setup E_t is proportional to $1/R^2$. Two situations are of interest. If $E_{rb} \ll E_t$, the dc term in (4) will be proportional to $1/R^4$ while the peak-to-peak variation will be proportional to $1/R^2$. If $E_{rb} \gg E_t$, the dc term will be independent of R but the peak-to-peak output will still vary as $1/R^2$. The detector outputs in the two situations are shown in Figs. 2(a) and 2(b) and correspond to the situations in Mode A and Mode B operation as described below.

Mode A: Dynamic Nulling of Undesired Signals

In the conventional static nulling procedure, the target is removed from the range and the reference signal amplitude and phase are adjusted to cancel the background signal [making $E_{rb}=0$ in (4)]. Then the target is reinserted and a reading of the received signal is taken. This procedure is satisfactory only if the presence of the target does not alter the background signal, requiring that the reflections from the target "shadow zone" constitute a small percentage of the over-all background signal. In general these requirements imply background level no greater than about 10 db above the target signal being measured.[11] The environment available to the writers was about 25 db worse than the apparent cross section of 0.05 square meter at a target distance of 25 feet which is considered satisfactory by Buckley.[12]

If the static nulling procedure is followed and the effect of target on background signal is negligible, the detector output (4) becomes

$$V_{\text{out}} = KE_t^2 \tag{5}$$

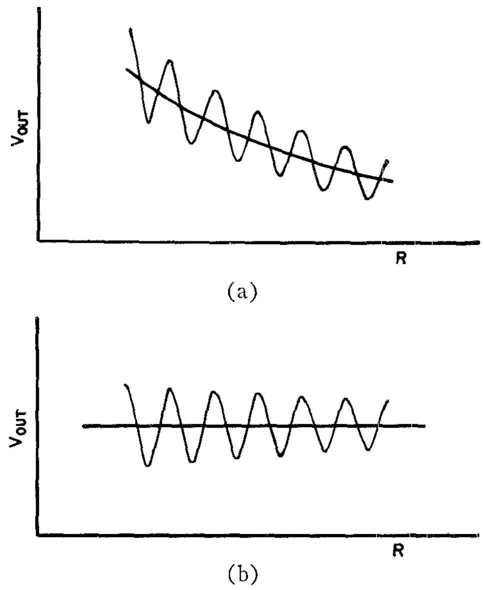

Fig. 2—Typical detector output voltages in the two modes of operation.

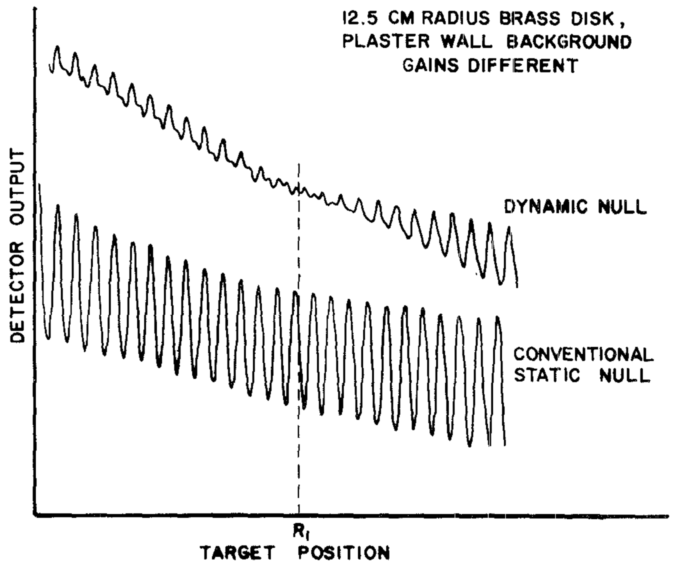

Fig. 3—Comparison of detector output of the dynamic null procedure to that of the conventional static null.

which is independent of target position. However, Fig. 3 shows the variation in detector output when the target does affect background signal.[13] The effects of target on equivalent background are apparent. Any static null measurement could be seriously in error.

If a dynamic null procedure is followed, this difficulty can be overcome. In this procedure the target is not removed from the range as it was in conventional nulling. Instead, the reference signal is adjusted in amplitude and phase to give a minimum peak-to-peak output voltage while the target is in motion. Fig. 3 shows the resulting detector output for the same target as for the

[11] E. F. Buckley, "Microwave reflectivity measurements, theory and practice," *Microwaves*, vol. 1, pp. 12–19; March 15, 1962. This value is from comments that the extraneous signal may be 30 db above minimum detectable signal and that measurements of cross sections lower than 20 db above minimum detectable signal should not be attempted in the conventional nulling procedure.

[12] E. F. Buckley, "The Design and Evaluation of Microwave Anechoic Chambers," Internal Rept. of Emerson and Cuming, Inc., Canton, Mass.; April, 1960.

[13] This was in an environment purposely chosen to have a high background signal; a plain plaster wall was in the background.

conventional static null. The equivalent background signal clearly varies with target position, and the null is therefore satisfactory at only one arbitrarily chosen target position R_1. At R_1 background cancellation is complete and the detector output is given by (5), allowing cross section to be read directly.

The "second harmonic" variation with target position evident near the null in Fig. 3 is caused by multiple reflections. Both the dynamic nulling and the overriding procedure discussed below have the advantage that multiple reflections cause a harmonic ripple in the output voltage which leads to their detection.

Not only does this dynamic null procedure compensate for background effects, but it also largely eliminates the problem of frequency drift of the transmitter. The acts of nulling and reading are performed simultaneously with the target in place.

Mode B: Overriding the Undesired Signals

The basic feature of the slowly moving target setup is the varying phase of the target return signal. In the dynamic null procedure, this was used to effect a cancellation of the nonvarying-phase part of the detector output. Without this cancellation (4) has a variational component proportional to target signal. Its dependence on the background signal would be of no consequence if both E_b and θ_b were constant. Even when nonconstant, however, if $E_r \gg E_b$ (4) becomes independent of background. Thus the detector output becomes as shown in Fig. 2(b), assuming $E_{rb} > E_t$. Neglecting the effects of range, R, the detector peak-to-peak output is

$$V_{pp} = 4KE_{rb}E_t. \quad (6)$$

With an audio attenuator calibrated in terms of 20 log V, relative back-scattering cross sections can be read directly by adjusting for equal oscilloscope peak-to-peak deflections. In the setup shown in Fig. 1 as applied to this mode of operation, the audio filter had a high-pass section to remove the "dc" terms and to remove some of the crystal detector excess noise mentioned in a later section.

This mode has the inherent advantages of the Doppler procedure in overcoming background. It is relatively insensitive to frequency drift although signal strength must remain constant long enough to compare measured targets to "standard" targets. Since it is, in effect, a CW system it suffers from crystal excess noise. It has the basic advantages of the dynamic null procedure and is much simpler. Since its operation is similar to synchronous detection,[14] its sensitivity is greater than that of the dynamic null procedure.

[14] M. E. Brodwin, C. M. Johnson, and W. M. Waters, "Low Level Synchronous Mixing," 1953 IRE CONVENTION RECORD, pt. 10, pp. 52–57.

OPERATING CHARACTERISTICS

The accuracy of the slowly moving target techniques is affected by several factors.

Range Effects

The problem of sufficient range to prevent excessive phase shift of the incident wave across the target cross section is the same here as in any back-scattering setup. In the work described, an effort was made to adhere to the customary $R \geq 2D^2/\lambda$.

As shown above, the variation of signal amplitudes with range does not affect the dynamic nulling procedure so long as the nulling and reading of the target signal are accomplished at the same value of R for the target being measured and the reference target. In the overriding technique, the peak-to-peak output is not exactly the value given by (6), since the target return does not have the same amplitude at the maximum and minimum points. In (4) it may easily be shown that the effects of range are small if adjacent maximum and minimum points of the output voltage are chosen. For typical values of $\lambda = 2.73$ cm, and the worst range condition ($R = 2D^2/\lambda$), the resulting error is about 0.1 per cent.

Sensitivity

In the dynamic null procedure the setup is, after nulling, simply functioning as a direct detection receiver (square law). Therefore the limits on sensitivity are the crystal excess noise and amplifier noise. In this mode of operation approximate sensitivity of the equipment described was -45 dbm allowing a back-scattering cross section of one square wavelength to be measured at a range of 150 cm when transmitting 400 mw at 11 Gc.

In the overriding procedure, (6) shows that a large signal output can be obtained by making reference signal E_r large so that E_{rb} becomes large. This, however, increases the crystal excess noise which varies approximately as crystal power squared.[15,16] The greatest signal-to-noise ratio, considering only crystal excess noise, is easily shown to occur for $E_{rb} \approx E_t$. One may be prevented from making this choice, however, by the necessity of choosing the reference signal large enough that E_{rb} is much larger than any variation in background signal. The background level then indirectly affects the sensitivity.

In this mode of operation the sensitivity of the equipment described was great enough to measure back-scattering cross sections as low as 20 db below a square

[15] J. M. Richardson and J. J. Faris, "Excess noise in microwave crystal diodes used as rectifiers and harmonic generators," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-5, pp. 208–212; July, 1957.

[16] M. W. P. Strandberg, H. R. Johnson, and J. R. Eshbach, "Apparatus for microwave spectroscopy," *Rev. Sci. Instr.*, vol. 25, p. 776; August, 1954.

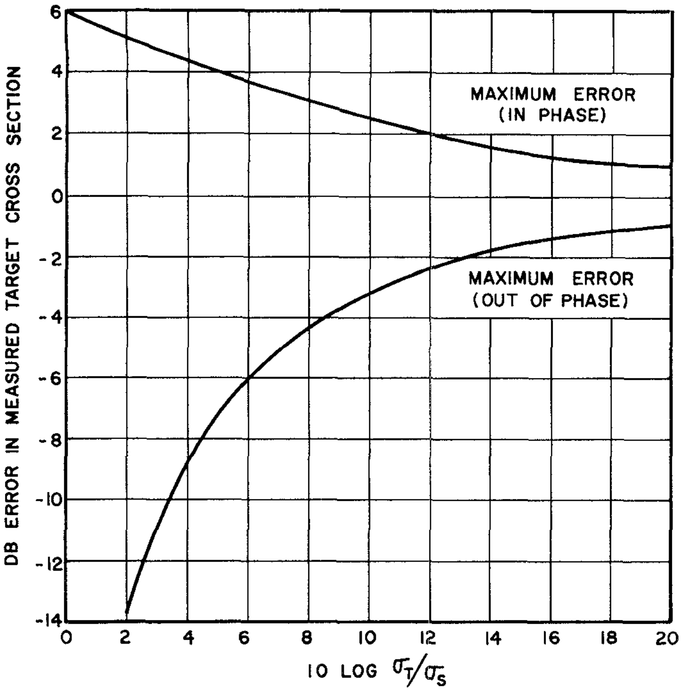

Fig. 4—Effects of target support cross section σ_s on the measured target cross section.

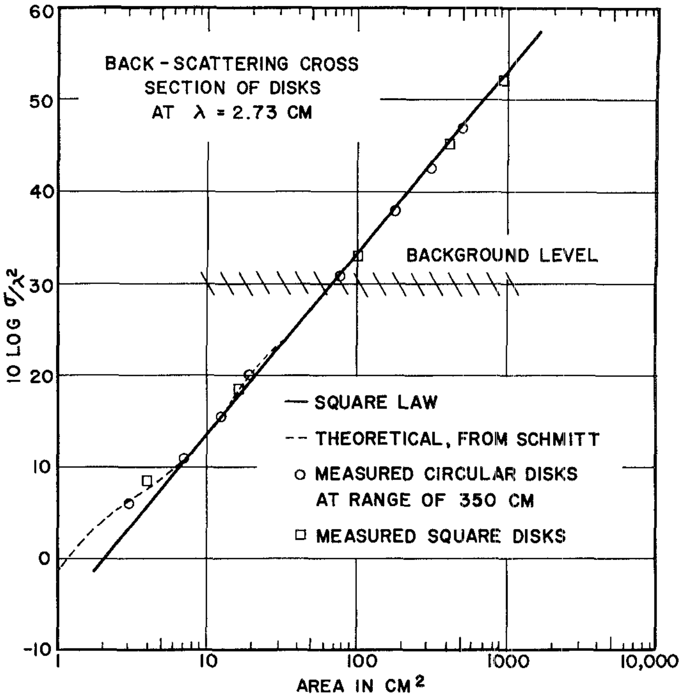

Fig. 5—Measured back-scattering cross sections of large disks.

wavelength at a range of 150 cm when transmitting 400 mw peak power at 11 Gc. By running the target back and forth and taking multiple exposures with an oscilloscope camera, it was possible to measure cross sections as low as 33 db below a square wavelength, under the same conditions. These correspond to receiver sensitivities of −65 dbm and −78 dbm, respectively.

Target Support

In both modes of operation, since it moves along with the target and thus produces a signal whose phase varies in the same way as the target signal, the support is not compensated for; thus its back-scatter cross section must be reduced to as small a value as possible. This could, of course, be eliminated by using a ground plane technique as in many Doppler setups, but this introduces additional problems. The possible error curve due to support scattering would be the same as that customarily used for background effects[17] if no interaction between target and target support were assumed. Fig. 4 shows the resulting maximum error possible and illustrates the rather stringent requirements on the support structure. In the measurements described here the support back-scattering cross section was reduced to 13 db below a square wavelength at 11 Gc by using polyfoam columns shaped to avoid specular reflection back to the antennas.

[17] W. P. Melling, "An Analysis of Radar Cross Section Measurement Techniques," Cornell Aeronautical Lab., Buffalo, N. Y., Rept. No. UB-1088-P-104; September, 1959.

Disk Measurements

The system was tested by comparing the measured results for thin disks with theory. No extensive use was made of the dynamic null procedure in actual measuring because of its poorer sensitivity when using the equipment of Fig. 1. The overriding technique was considered preferable and was tested extensively.

The back-scattering cross section of a large flat conducting disk is proportional to surface area A squared,[18]

$$\sigma_B = \frac{4\pi}{\lambda^2} A^2. \tag{7}$$

The cross section at smaller sizes is more complicated and depends on shape. The results of Schmitt[5] which he obtained from Andrejewski[19] were used for small circular disks. Fig. 5 shows measured values for both circular disks and rectangular disks compared to (7) and the Andrejewski theory. It should be noted that measurements were taken as low as 25 db below the background level (equivalent cross section at target position). The background was measured both by a direct comparison of transmitted and received signals and by observing the peak-to-peak detector output for several disks when the reference signal was zero (then the

[18] See, for example, R. F. Harrington, "Time-Harmonic Electromagnetic Fields," McGraw-Hill Book Co., Inc., New York, N. Y., Chapter 3; 1961.

[19] W. Andrejewski, "Die Beugung elektromagnetischer Wellen an der leitenden Kreisscheibe und anderen kreisfoermigen Oeffnungen im leitenden ebenen Schirm," thesis, Technische Hochschule, Aachen, Germany; 1952.

equivalent E_{rb} is actually E_b and a plot of peak-to-peak output vs target cross section σ_t has a maximum at $\sigma_b = \sigma_t$). The two methods agreed within a few db. In Fig. 5, the 2.0-cm radius disk was taken as a reference and the other cross sections plotted with respect to it.

These experimental results are considered to be quite satisfactory. It should be noted that they do not represent averages of readings but rather a given continuous run. In obtaining the data for Fig. 5 two values of reference signal E_r were used, one for disks of 2.0 cm radius and smaller and another for disks of 2.0 cm radius and larger.

Conclusion

Two simple procedures for measuring back-scattering cross sections have been presented. The essential feature of operation in each case is that the target moves slowly in a straight line toward the antennas. This slow motion need not be of constant velocity. The dynamic nulling procedure has an advantage over the conventional null procedure in that the null is accomplished with the target in place, thus reducing the time lag between nulling and measuring and eliminating the problem of target forward scatter affecting scattering from the background. However, the method does not compensate for the target support as does the conventional null. The overriding background procedure has the basic characteristic of the conventional Doppler procedure but the requirements on the target-moving system are greatly relaxed. In each instance, with the equipment described, the cross section is read directly from an adjustable attenuator. Since both procedures depend on observing a slow change in a signal level they are susceptible to low frequency noise, but signal averaging may be used. They have the insensitivity to background and transmitter drift of the conventional Doppler procedure. The two procedures described represent interesting innovations in the measuring of cross sections that can be very useful in certain cases, an example being the measuring of large odd-shaped objects in the presence of a poor background.